

Abstract

We present a metric theory of gravity with Lagrangian

$$L = (8\pi G)^{-1}(\Xi g^{ii} - \Upsilon g^{00})\sqrt{-g} + L_{GR} + L_{matter}$$

motivated by classical equations

$$\begin{aligned}\partial_t \rho + \partial_i(\rho v^i) &= 0 \\ \partial_t(\rho v^j) + \partial_i(\rho v^i v^j + p^{ij}) &= 0\end{aligned}$$

for a medium in Newtonian space-time. We obtain stable “frozen stars” instead of black holes and a “big bounce” instead of a big bang singularity.

A Metric Theory of Gravity with Condensed Matter Interpretation

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1 Introduction

There is a close analogy between condensed matter theory and gravity. It has been recognized that “effective gravity, as a low-frequency phenomenon, arises in many condensed matter systems” [14]. This has been used to study Hawking radiation and the Unruh effect [13] [12] [6] [14] and vacuum energy [14] for condensed matter examples. The general exchange of ideas with high energy physics, which “includes global and local spontaneous symmetry breaking, the renormalization group, effective field theory, solitons, instantons, and fractional charge and statistics” [16], is also worth to be mentioned.

The theory of gravity we present here suggests that this is not an accident, but the gravitational field is a medium in Newtonian space-time, described by usual condensed matter variables, with an interesting Lagrange formalism. Few general assumptions are sufficient to obtain a Lagrangian very close to GR, which fulfills the Einstein equivalence principle:

$$L = (8\pi G)^{-1}(\Xi g^{ii} - \Upsilon g^{00})\sqrt{-g} + L_{GR} + L_{matter}$$

After the derivation of the theory we consider quantization, some remarkable predictions, and compare the theory with other theories of gravity.

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2 The Theory

The theory describes a classical medium in a Newtonian framework – Euclidean space and absolute time. But we prefer to present the theory in a formalism where the non-covariant terms are disguised as covariant, with the preferred coordinates considered as usual scalar fields $X^\mu(x)$. It is easy to transform a non-covariant Lagrangian $L = L(T_{;\dots}, \partial_\mu T_{;\dots})$ into a (formally) covariant form $L = L(T_{;\dots}, \partial_\mu T_{;\dots}, X_{;\nu}^\mu)$.

The medium is described by steps of freedom typical for condensed matter theory. The gravitational field is defined by a positive density ρ , a velocity v^i , and a negative-definite symmetrical tensor field p^{ij} which we name “pressure”. The effective metric $g_{\mu\nu}$ is defined algebraically by

$$\begin{aligned}\hat{g}^{00} &= g^{00} \sqrt{-g} = \rho \\ \hat{g}^{i0} &= g^{i0} \sqrt{-g} = \rho v^i \\ \hat{g}^{ij} &= g^{ij} \sqrt{-g} = \rho v^i v^j + p^{ij}\end{aligned}$$

This decomposition of $g^{\mu\nu}$ into ρ , v^i and p^{ij} is a variant of the ADM decomposition. The signature of $g^{\mu\nu}$ follows from $\rho > 0$ and negative definiteness of p^{ij} .

The theory does not specify all properties of the medium, but only a few general properties – especially the conservation laws. The “material properties” of the medium, denoted by φ^m , remain unspecified. They are the matter fields. The complete specification – which includes the material laws of the medium – gives the theory of everything. The few general properties fixed here define a theory of gravity similar to GR. While it leaves the matter steps of freedom and the matter Lagrangian unspecified, it derives the Einstein equivalence principle.

In our covariant formalism the conservation laws may be defined as the Euler-Lagrange equations for the preferred coordinates. The related energy-momentum tensor

$$T_\mu^\nu = -\frac{\partial L}{\partial X_{;\nu}^\mu}$$

is not the same as in Noether’s theorem, but only equivalent. Now, we identify these conservation laws with the conservation laws we know from

condensed matter theory. First, the Euler-Lagrange equation for time we identify with the classical continuity equation for the medium:

$$\partial_t \rho + \partial_i(\rho v^i) = 0 \quad (1)$$

The equations for the spatial coordinates we identify with the Euler equation:

$$\partial_t(\rho v^j) + \partial_i(\rho v^i v^j + p^{ij}) = 0 \quad (2)$$

Note that we use here the identification of matter fields with material properties of the medium – we have no momentum exchange with external matter. The four conservation laws transform into the harmonic condition for the metric $g_{\mu\nu}$. Thus, they really look like equations for the preferred coordinates:

$$\square X^\nu = \partial_\mu(g^{\mu\nu} \sqrt{-g}) = 0$$

Therefore, we assume that the conservation laws are proportional to the Euler-Lagrange equations of $S = \int L$ for the preferred coordinates X^μ :

$$\frac{\delta S}{\delta X^\mu} \equiv -(4\pi G)^{-1} \gamma_{\mu\nu} \square X^\nu$$

We have introduced here a constant diagonal matrix $\gamma_{\mu\nu}$ and a common factor $-(4\pi G)^{-1}$ to obtain appropriate units. With Euclidean symmetry we obtain $\gamma_{11} = \gamma_{22} = \gamma_{33}$. Thus, we have two coefficients $\gamma_{00} = \Upsilon, \gamma_{ii} = -\Xi$. Now, the Lagrangian

$$L_0 = -(8\pi G)^{-1} \gamma_{\mu\nu} X_{,\alpha}^\mu X_{,\beta}^\nu g^{\alpha\beta} \sqrt{-g}$$

fulfils this property. For the difference $L - L_0$ we obtain

$$\frac{\delta \int (L - L_0)}{\delta X^\mu} \equiv 0$$

Thus, the remaining part is not only covariant in the weak sense, but does not depend on the preferred coordinates X^μ . But this is “strong” covariance, the classical requirement for the Lagrangian of general relativity. Thus, we can identify the difference with the classical Lagrangian of general relativity and obtain in the preferred coordinates

$$L = -(8\pi G)^{-1} \gamma_{\mu\nu} g^{\mu\nu} \sqrt{-g} + L_{GR}(g_{\mu\nu}) + L_{matter}(g_{\mu\nu}, \varphi^m)$$

with the following modification of the Einstein equations

$$G_{\nu}^{\mu} = 8\pi G(T_m)_{\nu}^{\mu} + (\Lambda + \gamma_{\kappa\lambda} g^{\kappa\lambda}) \delta_{\nu}^{\mu} - 2g^{\mu\kappa} \gamma_{\kappa\nu}$$

The last term is the full energy-momentum tensor, therefore, this equations defines a decomposition of the energy-momentum tensor into the energy-momentum tensor of matter and the energy-momentum tensor of the gravitational field defined by

$$(T_g)_{\nu}^{\mu} = (8\pi G)^{-1} (\delta_{\nu}^{\mu} (\Lambda + \gamma_{\kappa\lambda} g^{\kappa\lambda}) - G_{\nu}^{\mu}) \sqrt{-g}$$

3 Quantization

Most workers would agree that “at the root of most of the conceptual problems of quantum gravity” is the idea that “a theory of quantum gravity must have something to say about the quantum nature of space and time” [3]. These problems, especially the problem of time [5], simply disappear in a theory with fixed Newtonian background. Problems related with energy and momentum conservation too – the Hamiltonian is no longer a constraint.

The violation of Bell’s inequality is independent evidence for a preferred frame. A preferred frame is required for compatibility with the EPR criterion of reality [4] and Bohmian mechanics [2]. Bell himself concludes [1]: “the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincare thought that there was an aether — a preferred frame of reference — but that our measuring instruments were distorted by motion in such a way that we could no detect motion through the aether.”

Our theory is in ideal agreement with “the present educated view on the standard model, and of general relativity, ... that these are leading terms in effective field theories” [15] – an idea introduced by Sakharov [11]. It seems natural to assume that our medium has an atomic structure. An interpretation of ρ as the number of “atoms” per volume leads to an interesting prediction for the cutoff:

$$\rho(x)V_{cutoff} = 1.$$

It is non-covariant. For the homogeneous universe, it seems to expand together with the universe. It differs from the usual expectation that the cutoff is the Planck length $a_P \approx 10^{-33}cm$ (cf. [7], [14]).

4 Comparison with other theories of gravity

Because of the simplicity of the additional terms it is no wonder that they have been already considered. Two other theories have the same Lagrangian for appropriate signs of the cosmological constants: the “relativistic theory of gravity” proposed by Logunov et al. [9] and classical GR with some additional scalar “dark matter” fields. Nonetheless, equations are not all. There are other physical important things which makes the theories different as physical theories, like global restrictions, boundary conditions, causality restrictions, quantization concepts which are closely related with the underlying “metaphysical” assumptions.

4.1 Comparison with RTG

The “relativistic theory of gravity” (RTG) proposed by Logunov et al. [9] has Minkowski background metric $\gamma_{\mu\nu}$. The Lagrangian of RTG is

$$L = L_{Rosen} + L_{matter}(g_{\mu\nu}, \psi^m) - m_g^2 \left(\frac{1}{2} \gamma_{\mu\nu} g^{\mu\nu} \sqrt{-g} - \sqrt{-g} - \sqrt{-\gamma} \right)$$

which de facto coincides with our theory for $\Lambda = -m_g^2 < 0$, $\Xi = -\gamma^{11}m_g^2 > 0$, $\Upsilon = \gamma^{00}m_g^2 > 0$.

The metaphysical context of RTG is completely different. It is a special-relativistic theory, therefore incompatible with the EPR criterion of reality and Bohmian mechanics. Another difference is the causality condition: In RTG, only solutions where the light cone of g_{ij} is inside the light cone of γ_{ij} are allowed. A comparable but weaker condition exists in our theory too: $T(x)$ should be a time-like function, or, $\rho(X, T) > 0$. Note also that our theory suggests a different way of quantization: the prediction for the cutoff length l_{cutoff} is not Lorentz-covariant.

4.2 Comparison with GR plus dark matter

The Lagrangian is also equivalent to GR with some dark matter – four scalar fields X^μ . In this theory they are no longer preferred coordinates, but simply fields. Such “clock fields” in GR have been considered by Kuchar [8]. Usual energy conditions require $\Xi > 0, \Upsilon < 0$.

This GR variant allows a lot of solutions where the fields $X^\mu(x)$ cannot be used as global coordinates, especially solutions with non-trivial topology. They may also violate the condition that $X^0(x) = T(x)$ is time-like. Such solutions are forbidden in our theory. On the other hand, the infinite “boundary values” of the “fields” $X^\mu(x)$ are unreasonable for matter fields in GR. Another difference is that in our theory the X^μ are fixed background coordinates and therefore should not be quantized, while the “fields” $X^\mu(x)$ should be quantized.

5 Predictions

Using small enough values $\Xi, \Upsilon \rightarrow 0$ leads to GR equations. Therefore it is not problematic to fit observation. It is much more problematic to find a way to distinguish our theory from GR by observation.

5.1 A dark matter candidate

Let’s consider the influence of the new terms on the expansion of the universe. In our theory a homogeneous universe is flat. The the usual ansatz $ds^2 = d\tau^2 - a^2(\tau)(dx^2 + dy^2 + dz^2)$ gives

$$\begin{aligned} 3(\dot{a}/a)^2 &= -\Upsilon/a^6 + 3\Xi/a^2 + \Lambda + \varepsilon \\ 2(\ddot{a}/a) + (\dot{a}/a)^2 &= +\Upsilon/a^6 + \Xi/a^2 + \Lambda - p \end{aligned}$$

We see that Ξ influences the expansion of the universe similar to dark matter with $p = -\frac{1}{3}\varepsilon$.

5.2 Big bounce instead of big bang singularity

Υ becomes important only in the very early universe. But for $\Upsilon > 0$, we obtain a qualitatively different picture. We obtain a lower bound a_0 for $a(\tau)$ defined by

$$\Upsilon/a_0^6 = 3\Xi/a_0^2 + \Lambda + \varepsilon$$

The solution becomes symmetrical in time, with a big crash followed by a big bang. For example, if $\varepsilon = \Xi = 0$, $\Upsilon > 0$, $\Lambda > 0$ we have the solution

$$a(\tau) = a_0 \cosh^{1/3}(\sqrt{3\Lambda}\tau)$$

In time-symmetrical solutions of this type the horizon is, if not infinite, at least big enough to solve the cosmological horizon problem (cf. [10]) without inflation.

5.3 Frozen stars instead of black holes

The choice $\Upsilon > 0$ influences also another physically interesting solution – the gravitational collapse. There are stable “frozen star” solutions with radius slightly greater than their Schwarzschild radius. The collapse does not lead to horizon formation, but to a bounce from the Schwarzschild radius. Let’s consider an example. The general stable spherically symmetric harmonic metric depends on one step of freedom $m(r)$ and has the form

$$ds^2 = (1 - \frac{m}{r} \frac{\partial m}{\partial r}) (\frac{r-m}{r+m} dt^2 - \frac{r+m}{r-m} dr^2) - (r+m)^2 d\Omega^2$$

Let’s consider the ansatz $m(r) = (1 - \Delta)r$. We obtain

$$\begin{aligned} ds^2 &= \Delta^2 dt^2 - (2 - \Delta)^2 (dr^2 + r^2 d\Omega^2) \\ 0 &= -\Upsilon \Delta^{-2} + 3\Xi(2 - \Delta)^{-2} + \Lambda + \varepsilon \\ 0 &= +\Upsilon \Delta^{-2} + \Xi(2 - \Delta)^{-2} + \Lambda - p \end{aligned}$$

Now, for very small Δ even a very small Υ becomes important, and we obtain a non-trivial stable solution for $p = \varepsilon = \Upsilon g^{00}$. Thus, the surface remains visible, with time dilation $\sqrt{\varepsilon/\Upsilon} \sim M^{-1}$.

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